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MODELING AND ANALYSIS OF VOLUMETRIC THERMAL ERROR IN TWO AXIS HORIZONTAL TURNING CENTER USING RIGID BODY KINEMATICS

Dr.S.Ramesh Babu*[1], Mr.M.Mohan[2], Mr.S.Dhamotharan[3]

*[1]Professor., Dept. of Mechanical Engg., KPR Institute of Engineering & Technology, Coimbatore 641 407.

[2,3]Asst. Prof., Dept. of Mechanical Engg., Dhanalakshmi Srinivasan College of Engg., Coimbatore 641 105.

ABSTRACT

Quality of the products manufactured is the basic necessity in global manufacturing system. Machine tools must be periodically serviced to perform towards attaining higher productivity with better quality. Periodic maintenance, calibrations, identification of volumetric displacement of the components of the machine tool due to wear and tear, thermal behaviors, etc., are most essential. In this work the geometric dislocations of the components of a two axes CNC lathe at various positions on the machine tool are cumulatively summed and the total volumetric error behavior at the tool center point(TCP) is plotted in 3D. Experiments were carried out for more than twenty hours for a particular load cycle and the geometric and thermal errors were identified at each stage. An expression for total volumetric error is derived by synthesizing the machine's parametric errors such as linear position errors, pitch, roll, yaw, etc., using homogeneous matrix transformations. With the derived mathematical model the error behavior/volume at the tool-work piece interface is plotted and compared with the experimental results.

***Keywords**: Volumetric displacements, geometric and thermal errors, homogeneous matrix transformations.*

I. INTRODUCTION

Machine tools are the most important means of production for the metal-working industries[1]. In a typical machine tool, there are multiple error origins[2] including geometric, static and dynamic loading, thermal, mismatches between servo-loop parameters, interpolation etc. Geometric errors[3] of machine tools come from manufacturing defects, machine wear and static deflection of machine components. Thermal errors[3] results from thermal distortions of machine components due to internal heat sources, such as motors, bearing, hydraulic system and ambient temperature. Thermally induced error play a major role in the accuracy of processes on machine tools. Eung-Suk Lee, Suk-Hwan Sub and Jin-Wook Shon[4] has investigated that the geometric error of a machined part is due not only to the volumetric error of the machine tool but also due to in-process errors and environmental errors. Environmental errors, which can be mainly represented by the thermal distortion and expansion due to temperature changes and heat flux, do not affect the accuracy of a machined part independently. This means that environmental errors always emerge in combination with other errors. Temperature change is a very important factor in modeling and compensation of volumetric error. S.-H. Yang K.-H. Kim Y. K. Park S.-G. Lee, [5] has stated that the machining accuracy is directly influenced by the quasi-static errors of a machine tool. Quasi-static errors are defined as the relative position errors between the work piece and the tool. Geometric errors and thermal errors of a machine tool belong to quasi-static error which is associated with the structure of the machine tool itself. According to C.Raksiri, and M.Panrnichkum[6], the kinematic and geometric error, are the basic inaccuracies of the machine tool. For a three axes machine, there are 21 error components (3 linear position errors, 6 straightness errors, 9 angular error and 3 squareness errors) the derivation of the kinematic and geometric error is based on the assumption of rigid body motions.

A. K. Srivastava, s. C. Veldhuist and m. A. Elbestawlt[7] have tried a systematic approach for the development of geometrical and thermal error based on the kinematic analysis of the five axis CNC machine tool. The development of the model shows that angular deviations are independent of translational errors. However, the tool point deviations are dependent on both translational and rotational errors[8]. The model has been used for the design and testing of a compensation strategy. The simulation studies indicate that CNC compensation for errors in X, Y and Z axes are possible. However, the capability of the CNC compensation for pitch, roll and yaw errors is dependent on

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the positioning of the rotary axes on the machine tool. A.C. Okafor *, Yalcin M. Ertekin[9] has presented the derivation of a general volumetric error model, which synthesizes both geometric and thermal errors of the VMC using homogenous matrix transformations of the axis slides.

In this research, derivation of a mathematical volumetric error model is derived from the homogeneous coordinate transformation method which synthesizes both geometric and thermal errors of two axis horizontal turning machine. The developed volumetric error model describe the error structure of a machine tool and relates these error in the relative motion between tool and workpiece with respect to machine coordinate frame

II. OVERVIEWOF TWO AXES HORIZONTAL CNC LATHE ERRORS

In general, elements that contribute to the total error of machining center are bed, column, spindle and its slides and the various linear and rotary axes. The total errors are the errors due to geometric inaccuracies and thermally induced errors.

2.1 Errors due to geometric inaccuracies

Due to expansion and contraction of the machine elements, misalignment leading to the mechanical imperfection occurs. This causes the geometric errors between tool and work piece, as position and orientation errors. The effect of the geometric inaccuracies results in producing errors in the carriage, cross slide, turret and spindle.

2.2 Thermally induced errors

In all the elements of the machine tool, carriage, cross slide, turret and spindle the errors occur in all the six degrees of freedom. These six degrees of freedom errors can be taken as three translation errors and three rotation errors (pitch, roll, yaw). Fig. 1 shows the six error components of a turning center's Z-axis carriage system.

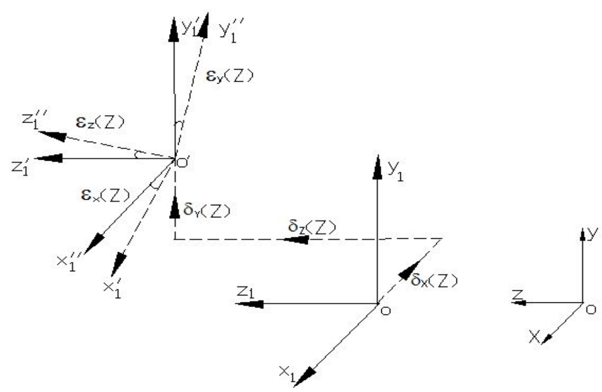

Fig. 1 Six error components at a point (O)

In the above figure, OXYZ is the reference coordinate system, $O_1X_1Y_1Z_1$ is the ideal position of carriage coordinate system, $O_1'X_1'Y_1'Z_1'$ is the actual position of carriage coordinate system (for translational errors alone) and $O_1''X_1''Y_1''Z_1''$ is the actual position of carriage coordinate system (with both translational and rotational errors) with Z being the direction of motion. The representations of the positional errors are as listed.

εx(z) is the rotational error about X-axis (pitch)
εy(z) is the rotational error about Y-axis (yaw)
εz(z) is the rotational error about Z-axis (roll)
δx(z) is the translational error along Z axis
δy(z) is the translational error along X axis
δz(z) is the translational error along Y axis

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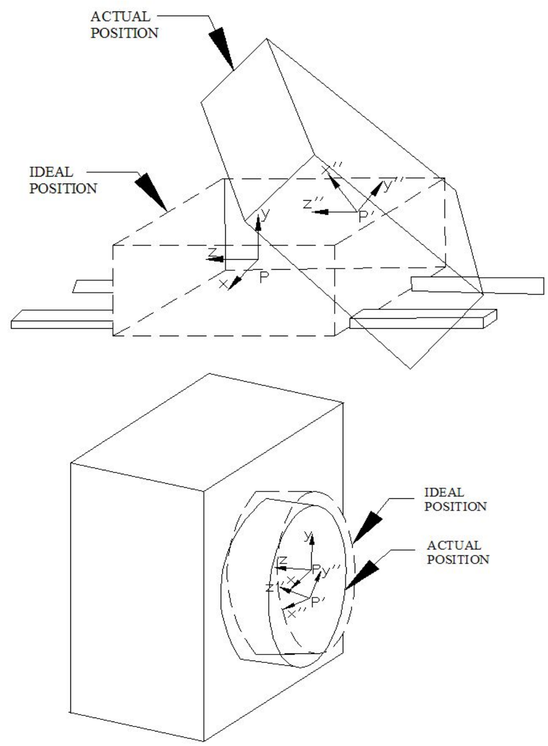

Fig. 2 Translation and rotation error components

The translational and rotational errors at various positions are demonstrated in Fig 2. The total error motion is the combination of rotation and translation effects.

III. HOMOGENOUS TRANSFORMATION MATRIX

A homogeneous coordinate transformation system transforms a rigid body position and direction from one coordinate system to another coordinate system. Homogeneous transformation matrix allows arbitrary linear transformations to be represented in a consistent format, suitable for compensation. The 3 dimensional space representation can be represented in 4×4 matrix as in equation (1).

$$T_x = \begin{bmatrix} R_{3\times 3} & P_{3\times 1} \\ f_{1\times 3} & 1\times 1 \end{bmatrix} \quad \text{……….(1)}$$

where

$R_{3\times3}$ – Rotation Matrix
$P_{3\times1}$ – Position Vector
$f_{1\times3}$ – Perspective transformation
1×1 – Scaling

3.1 Linear transformations

When a point moves in space, its location can be identified by the distance travelled with reference to an origin of a coordinate system. The position of the point can be expressed as a vector that can be expressed as the first three elements of the 4th column of the 4 x 4 homogenous matrix (equation 2). In three axes NC machine tools, if there are three linear axes, tool tip is expressed as a distance travelled from the reference point.

$$T_{rans} = Trans(x,y,z) = \begin{bmatrix} 1 & 0 & 0 & \delta x(X) \\ 0 & 1 & 0 & \delta y(X) \\ 0 & 0 & 1 & \delta z(X) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots\ldots(2)$$

where δx(X), δy(X) and δz(X) are the nominal distance along x, y and z-axes corresponding to the X axes of machine tool. The vector P has three components in three dimension of the coordinate system.

3.2 Rotation transformations

When a tool moves around an axis its direction and tool tip position can be uniquely defined by the angle rotated around the axis. For three-axis machine tool, position is described by the three linear movements. For four or five axes machine tools, tool tip and direction is obtained by forward kinematics.

The transformation corresponding to rotations about the X, Y and Z-axes by an angle θ are:

$$Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots\ldots (3)$$

$$Rot(y,\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots\ldots (4)$$

$$Rot(z,\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \ldots\ldots\ldots (5)$$

3.3 Composite Rotation Matrix

$$T_{Rot} = Rot(x,X) \times Rot(y,X) \times Rot(z,X)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -Ex(X) & 0 \\ 0 & Ex(X) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & Ey(X) & 0 \\ 0 & 1 & 0 & 0 \\ -Ey(X) & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -Ex(X) & 0 & 0 \\ -Ex(X) & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{Rot} = \begin{bmatrix} 1 & -Ez(X) & Ey(X) & 0 \\ Ez(X) & 1 & -Ex(X) & 0 \\ -Ey(X) & Ex(X) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \ldots\ldots\ldots(6)$$

Transformation matrix Tx

$$Tx = TRot \times TTrans$$

$$= \begin{bmatrix} 1 & -Ez(X) & Ey(X) & 0 \\ Ez(X) & 1 & -Ex(X) & 0 \\ -Ey(X) & Ex(X) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & \delta x(X) \\ 0 & 1 & 0 & \delta y(X) \\ 0 & 0 & 1 & \delta z(X) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_x = \begin{bmatrix} 1 & -Ez(X) & Ey(X) & \delta x(X) \\ Ez(X) & 1 & -Ex(X) & \delta y(X) \\ -Ey(X) & Ex(X) & 1 & \delta z(X) \\ 0 & 0 & 0 & 1 \end{bmatrix} \ldots\ldots.. \quad (7)$$

Similarly Transformation matrix Ty

$$T_Y = \begin{bmatrix} 1 & -Ez(Y) & Ey(Y) & \delta x(Y) \\ Ez(Y) & 1 & -Ex(Y) & \delta y(Y) \\ -Ey(Y) & Ex(Y) & 1 & \delta z(Y) \\ 0 & 0 & 0 & 1 \end{bmatrix} \ldots\ldots.(8)$$

Similarly Transformation matrix Tz

$$Tz = \begin{bmatrix} 1 & -Ez(Z) & Ey(Z) & \delta x(Z) \\ Ez(Z) & 1 & -Ex(Z) & \delta y(Z) \\ -Ey(Z) & Ex(Z) & 1 & \delta z(Z) \\ 0 & 0 & 0 & 1 \end{bmatrix} \ldots\ldots\ldots(9)$$

IV. DERIVATION OF ERROR MODELS FOR 2 AXIS CNC LATHE

The volumetric errors can be innovatively realized in terms of reference as the nominal position of any point in space is specified with a common reference.The references are the machine origin which are functional over the working zone of the machine. The actual position and orientation of the Z-axis carriage (Fig.4) in reference coordinate system (machine zero) is given as follows

$${}^{R}T_1 = \begin{bmatrix} 1 & -\varepsilon_{z1}(z) & \varepsilon_{y1}(z) & \delta_{x1}(z)+a_1 \\ \varepsilon_{z1}(z) & 1 & -\varepsilon_{x1}(z) & \delta_{y1}(z)+b_1 \\ -\varepsilon_{y1}(z) & \varepsilon_{x1}(z) & 1 & \delta_{z1}(Z)+z+c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

where

$_{y1}(z)$ is the roll error of Z-axis
$_{x1}(z)$ is the pitch error of Z-axis
$_{z1}(z)$ is the yaw error of Z-axis
a_1 is the constant offset in X direction between O and O_1
b_1 is the constant offset in Y direction between O and O_1
c_1 is the constant offset in Z direction between O and O_1
$_{y1}(z)$ is the linear displacement error of Z axis
$_{x1}(z)$ is the X straightness of Z-axis as it moves in Z direction
$_{z1}(z)$ is the Z straightness of Z-axis as it moves in Z direction

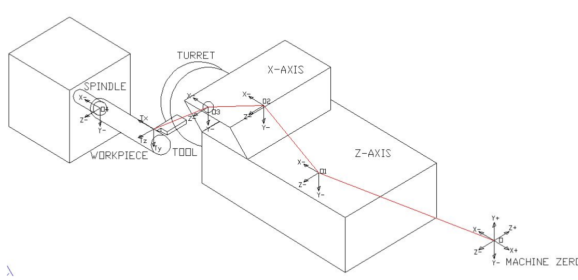

Fig-4 transformation in the tool point's coordinate frame to reference coordinate frame

The actual position and orientation of the X-axis cross slide carriage of the CNC Lathe (Fig. 4) in Z-axis carriage coordinate system is given as follows:

$$ {}^{1}T_{2} = \begin{bmatrix} 1 & -\varepsilon_{z2}(x) & \varepsilon_{y2}(x) & \delta_{x2}(x)+a_{2}+x \\ \varepsilon_{z2}(x) & 1 & -\varepsilon_{x2}(x) & \delta_{y2}(x)+b_{2} \\ -\varepsilon_{y2}(x) & \varepsilon_{x2}(x) & 1 & \delta_{z2}(x)+c_{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11) $$

where
$_{x2}$(x) Roll error of X-axis
$_{y2}$(x) Pitch error of X-axis
$_{z2}$(x) Yaw error of X-axis
a_2 Constant offset in X direction between O1 and O_2
b_2 Constant offset in Y direction between O1 and O_2
c_2 Constant offset in Z direction between O1 and O_2
$_{x2}$(x) linear displacement error of X-axis
$_{y2}$(x) Y straightness of X-axis as it moves in X direction
$_{z2}$(x) Z straightness of X-axis as it moves in X direction

The actual position and orientation of the Z-axis turret rotation of the CNC Lathe (Fig. 4) in X-axis cross slide carriage coordinate system is given as follows:

$$ {}^{2}T_{3} = \begin{bmatrix} 1 & -\varepsilon_{z3}(zt) & \varepsilon_{y3}(zt) & \delta_{x3}(zt)+a_{3} \\ \varepsilon_{z3}(zt) & 1 & -\varepsilon_{x3}(zt) & \delta_{y3}(zt)+b_{3} \\ -\varepsilon_{y3}(zt) & \varepsilon_{x3}(zt) & 1 & \delta_{z3}(Zt)+c_{3} \\ 0 & 0 & 0 & 1 \end{bmatrix} \ldots (12) $$

where
$_{z3}$(*zt*) is the roll error of Z-axis turret rotation
$_{x3}$(*zt*) is the pitch error of Z-axis turret rotation
$_{y3}$(*zt*) is the yaw error of Z-axis turret rotation
a_3 is the constant offset in X direction between O_2 and O_3
b_3 is the constant offset in Y direction between O_2 and O_3
c_3 is the constant offset in Z direction between O_2 and O_3
$_{z3}$(*zt*) is the linear displacement error of Z axis
$_{x3}$(*zt*) is the X straightness of Z-axis as it moves in Z direction turret rotation
$_{y3}$ (*zt*) is the Y straightness of Z-axis as it moves in Z direction turret rotation

The actual position and orientation of the Z-axis spindle rotation (Fig. 5) in reference coordinate system is given as follows

$$ {}^{R}T_{4} = \begin{bmatrix} 1 & -\varepsilon_{z4}(zw) & \varepsilon_{y4}(zw) & \delta_{x4}(zw)+a_{4} \\ \varepsilon_{z4}(zw) & 1 & -\varepsilon_{x4}(zw) & \delta_{y4}(zw)+b_{4} \\ -\varepsilon_{y4}(zw) & \varepsilon_{x4}(zw) & 1 & \delta_{z4}(Zw)+c_{4} \\ 0 & 0 & 0 & 1 \end{bmatrix} \ldots (13) $$

where
$_{z3}$(*zw*) is the roll error of Z-axis spindle rotation
$_{x3}$(*zw*) is the pitch error of Z-axis spindle rotation
$_{y3}$(*zw*) is the yaw error of Z-axis spindle rotation

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a_3 is the constant offset in X direction between O and O_4
b_3 is the constant offset in Y direction between O and O_4
c_3 is the constant offset in Z direction between O and O_4
$_{z3}(zw)$ is the linear displacement error of Z axis spindle rotation
$_{x3}(zw)$ is the X straightness of Z-axis as it moves in Z direction spindle rotation
$_{y3}(zw)$ is the Y straightness of Z-axis as it moves in Z direction spindle rotation

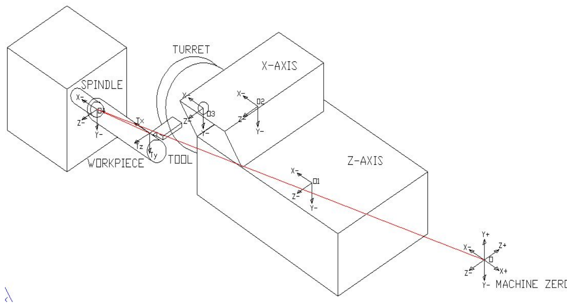

Fig. 5 Transformation in the workpiece point's coordinate frame to reference coordinate frame

The cutting edge of the cutting tool, which is attached to the spindle be ${}^{3}T_{Tool}$ and assume a vector ${}^{4}T_{Work}$ represents the cutting edge at the work piece which is fixed on the worktable (Fig. 6). Using homogenous transformation matrices these two cutting edges can be transformed to the references (R) frame. There are no linkage errors in the ideal case. As a result, these two cutting edges must be coincident. Therefore, we have:

……… (14)

$${}^{R}T_{Work} = {}^{R}T_4 \times {}^{4}T_{Work} \quad \text{………... (15)}$$

$${}^{R}T_{Tool} = {}^{R}T_{Work} \quad \text{………... (16)}$$

$${}^{R}T_{Tool} = {}^{R}T_1 \times {}^{1}T_2 \times {}^{2}T_3 \times {}^{3}T_{Tool}$$

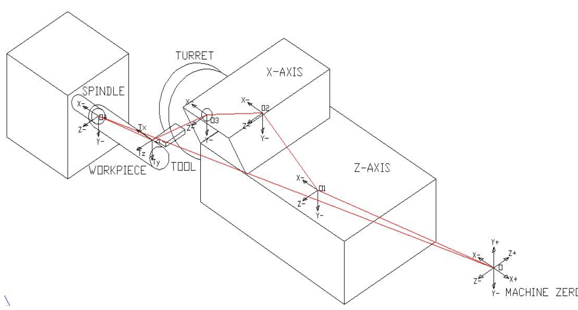

Fig. 6 Transformation in the tool point's coordinate frame to reference coordinate frame and the transformation in the workpiece point's coordinate frame to reference coordinate frame

${}^{3}T_{Tool}$ and ${}^{4}T_{Work}$ are represented by the following vectors in their respective coordinate frames

$${}^{3}T_{Tool} = [T_x \quad T_y \quad T_z \ldots 1]^T \ldots \text{(17)}$$

$${}^{4}T_{Wool} = [W_x \quad W_y \quad W_z \quad 1]^T \ldots\ldots \text{(18)}$$

where
Wx, Wy and Wz are workpiece coordinates; and Tx, Ty, and Tz are the X, Y and Z offsets of the tool tip, respectively. We use T to indicate transpose of the matrix. For the ideal case, transformation matrices of $^RT_1$, $^1T_2$, $^2T_3$ and $^RT_4$ are given as follows:

$$^RT_1 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & b_1 \\ 0 & 0 & 1 & z+c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{........(19)}$$

$$\text{.........(20)}$$

$$^2T_3 = \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 1 & 0 & b_3 \\ 0 & 0 & 1 & c_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{........(21)}$$

$$^RT_4 = \begin{bmatrix} 1 & 0 & 0 & a_4 \\ 0 & 1 & 0 & b_4 \\ 0 & 0 & 1 & c_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{.........(22)}$$

where
a_1, b_1, c_1 are constant offsets between O and O_1;
a_2, b_2, c_2 are constant offsets between O_1 and O_2;
a_3, b_3, c_3 are constant offsets between O_2 and O_3;
a_4, b_4, c_4 are constant offsets between O and O_4;
x and y are the nominal axis positions.

After carrying out the matrix multiplication, the workpiece coordinate vector can be calculated as follows:
$^2T_{Work}= [^RT_1{}^1T_2]\ \ ^1[^RT_3{}^3T_{Tool}]$.........(23)

$$^3T_{Tool} = \begin{bmatrix} -a1-a2-a3+a4-x+Wx \\ -b1-b2-b3+b4+Wy \\ -c1-c2-c3+c4+zw+Wz \\ 1 \end{bmatrix} \quad \text{.......(24)}$$

However, the linkages are not pretend to be perfect,24 parametric errors will cause a relative error between tool and workpiece.So, the spatial relationship between the cutting tool and a point on the workpiece actually expressed as:

$^RT_{Work}= {}^RT_{Tool}$ (E_V ....(25)
$E_V= {}^RT^{-1}_{Tool}$ ($^RT_{Work}$ ....(26)

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where E_V is the volumetric error homogeneous transformation matrix representing position and orientation errors between the cutting tool and workpiece. The position vector component of E_V represents the translations in the tool point's coordinate frame that must be made to the tool point in order to be at the proper location on the workpiece.

Inverse kinematic solutions should be used for the implementation of error correction algorithms in machining with revolution and translational axes.Volumetric error, E_V can be calculated using the position vectors of the homogenous transformation matrices. The error correction vector ${}^{R}P_{Correction}$ with respect to the reference coordinate frame can be obtained from the following matrix equation:

$$ {}^{R}P_{Correction} = {}^{R}\begin{bmatrix} Px \\ Py \\ Pz \end{bmatrix}_{Correction} = {}^{R}\begin{bmatrix} Px \\ Py \\ Pz \end{bmatrix}_{Tool} - {}^{R}\begin{bmatrix} Px \\ Py \\ Pz \end{bmatrix}_{Work} $$

The matrix equation for error correction vector
${}^{R}P_{Correction}$ to the reference coordinate frame can be represented as

$$ {}^{R}P_{Correction} = {}^{R}P_{Tool} - {}^{R}P_{Work} $$
$$ = P({}^{R}T_4 \times {}^{4}T_{Work})_{actual} - P({}^{R}T_1 \times {}^{1}T_2 \times {}^{2}T_3 \times {}^{3}T_{Tool})_{actual} $$

${}^{R}P_{Correction}$ will not necessarily be equal to position vector component of Ev due to offsets and angular orientation errors of the axes. The incremental motions of X, Y and Z-axes which represented by ${}^{R}P_{Correction}$ will compensated for tool point position errors on MNC.Using Mathematica software the expanded form of the matrix is obtained as follows:

$$ Px=a1+a2-a4-Wx+x+\delta x1+\delta x2-\delta x4+((c2+\delta z2)\varepsilon y1-Wz\varepsilon y4)((b2+\delta y2)\varepsilon z1)+(c3+\delta z3)(\varepsilon y1+\varepsilon y2)+(b3+\delta y3)(-\varepsilon z1-\varepsilon z2)+(a3+\delta x3)+(-c1-c2-c3+c4+Wz-z)(\varepsilon y1+\varepsilon y2-\varepsilon x3(-\varepsilon z1-\varepsilon z2)+\varepsilon y3+(-a1-a2-3+a4+Wx-x)(1-\varepsilon y3(\varepsilon y1+\varepsilon y2)+(-\varepsilon z1-\varepsilon z2)\varepsilon z3)+(-b1-b2-b3+b4+Wy)(-\varepsilon z1+\varepsilon x3(\varepsilon y1+\varepsilon y2)-\varepsilon z2-\varepsilon z3)+Wy\varepsilon z4 \quad \ldots\ldots\ldots(27) $$

$$ Py=b1+b2-b4-Wy+\delta y1+\delta y2-\delta y4-(c2+\delta z2)\varepsilon x1+(Wz\varepsilon x4)+(a2+x+\delta x2)\varepsilon z1)+(c3+\delta z3)(-\varepsilon x1-\varepsilon x2)+(a3+\delta x3)(\varepsilon z1+\varepsilon z2)+(b3+\delta y3)+(-c1-c2-c3+c4+Wz-z)(\varepsilon y3(\varepsilon z1+\varepsilon z2)-\varepsilon x3+((-b1-b2-b3+b4+Wy)(1+\varepsilon x3(-\varepsilon x1-\varepsilon x2)-(\varepsilon z1+\varepsilon z2)\varepsilon z3))+(-a1-a2-a3+a4+Wx-x)(\varepsilon z1-\varepsilon y3(-\varepsilon x1-\varepsilon x2)+\varepsilon z2+\varepsilon z3)-Wx\varepsilon z4 \quad \ldots\ldots\ldots(28) $$

$$ Pz=c1+c2-c4-Wz+z+\delta z1+\delta z2-\delta z4+(b2+\delta y2)\varepsilon x1-(Wy\varepsilon x4)(a2+x+\delta x2)\varepsilon y1+(c3+\delta z3)+Wx\varepsilon y4+(a3+\delta x3)(-\varepsilon y1-\varepsilon y2)+(b3+\delta y3)(\varepsilon x1+\varepsilon x2)+(-c1-c2-c3+c4+Wz-z)+\varepsilon y3(-\varepsilon y1-\varepsilon y2)-\varepsilon x3(\varepsilon x1+\varepsilon x2)+(-b1-b2-b3+b4+Wy)(\varepsilon x1+\varepsilon x2+\varepsilon x3-(-\varepsilon y1-\varepsilon y2)\varepsilon z3)+(-a1-a2-a3+a4+Wx-x)(-\varepsilon y1-\varepsilon y2-\varepsilon y3+(\varepsilon x1+\varepsilon x2)\varepsilon z3 \quad \ldots\ldots\ldots(29) $$

Px, Py and Pz are the volumetric error compensation components in X, Y and Z directions respectively. The resultant volumetric error can be obtained by the following equation

$$ {}^{E}R_V=(P_x^2+P_y^2+P_z^2)^{1/2} \quad \ldots\ldots\ldots(30) $$

where ${}^{E}R_V$ is the resultant volumetric error and Px, Py and Pz are the volumetric error components in X, Y and Z directions respectively. X, Y and Z-axes volumetric error components of the two axis CNC Lathe were calculated by using volumetric error model.

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X-Y-Z axes volumetric components for particular loads were compensated in the equation 28,29,30. The result for the compensated loads were obtained in the form of graph.

The machine run is withheld for 2-8 hours in non-load conditions, the results were obtained. The spindle, turret rotation positional errors are also obtained.The spindle rotation displacement (Fig.7) shows that the z-axis displacement is maximum under cold start condition and with the time extension the deviation in all the axes tends to be normal.

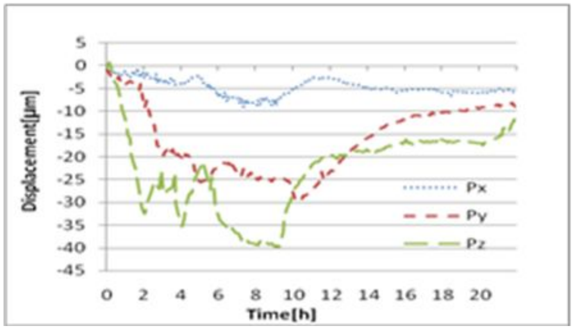

Fig.7 Displacement spindle rotation

In the z-axis load displacement (Fig.8), y-axis and z-axis displacements are maximum and tend to be in-line over a period of time.

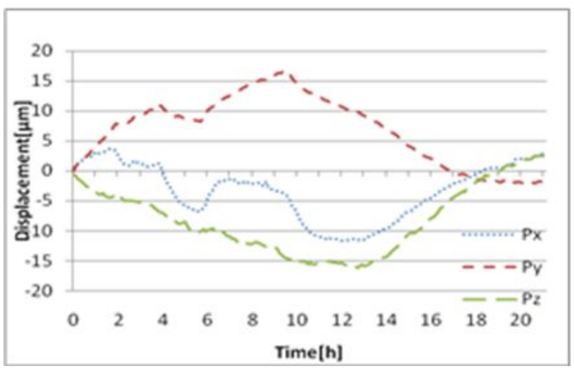

Fig. 8 Displacement Z -axis load

x-axis load displacement (Fig.9) the z-axis deviation gets oscillated frequently in both positive and negative terms of reference.

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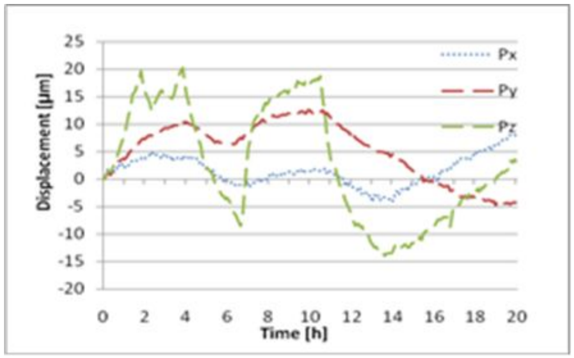

Fig. 9 Displacement X-axis load

The displacement Turret rotation (Fig.10) reveals, there is no line-in-reference in any axes as they deviate and maintain the same displacement even as the time extends.

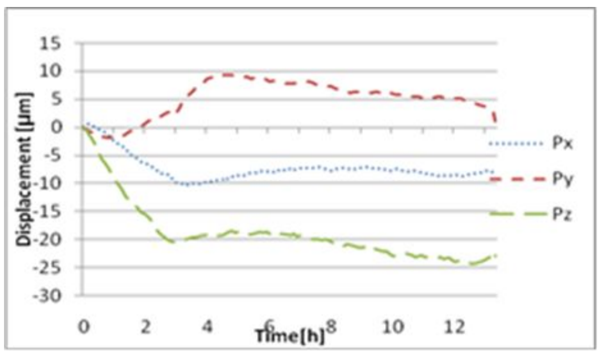

Fig. 10 Displacement Turret rotation

The volumetric error for x, y, z axes are combined (fig.11) into a single visual which shows the 3 axes displacement placed in-line reference after a period of time and again get deviated from in-line reference.

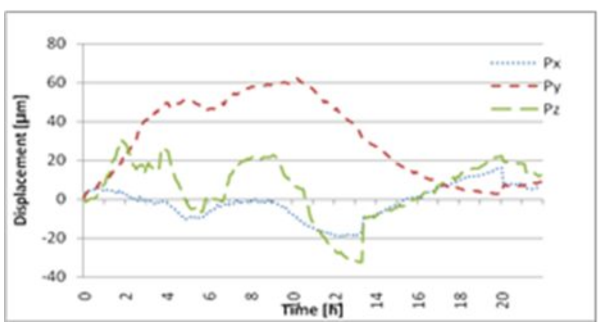

Fig. 11 Volumetric error component

The resultant x, y and z axes displacement graph (fig.12) obtained from the resultant volumetric equation (31) shows maximum displacement of

65µm under cold start condition and over the time extension it gets reduced.

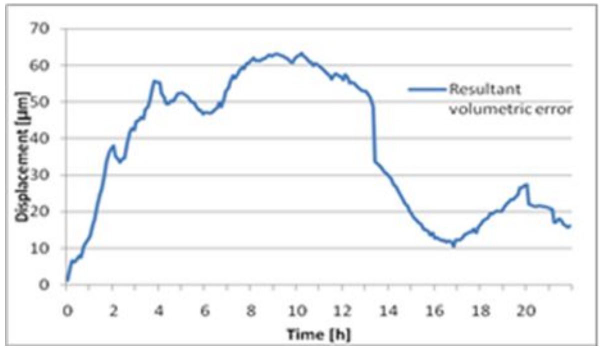

Fig.12 Resultant volumetric error

The final 3D error plot (Fig.13) for resultant volumetric error is obtained

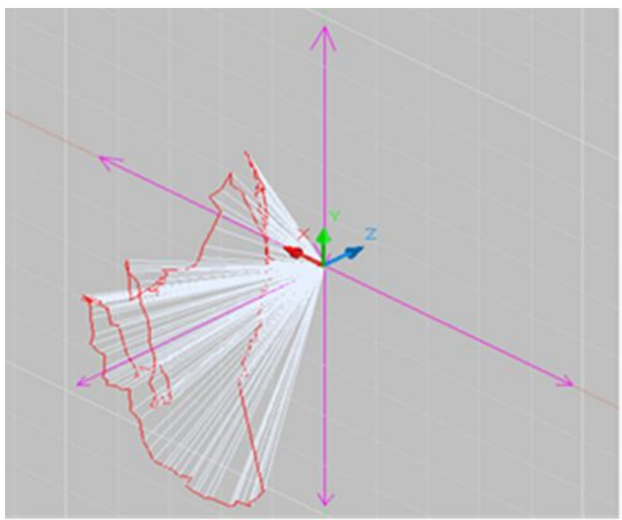

Fig. 13 Resultant volumetric error plot in 3D

V. CONCLUSION

The following conclusions have been derived successfully by this research.

- By the rigid body kinematics, the horizontal turning center axis are modeled and a homogeneous transformation matrix are derived.
- The volumetric error compensation for geometric and positional error vector has been successfully established.
- As the final figure (Fig.12) shows that there was peak displacement during the cold start conditions, initial 2 hours non-load run of machine is required to compensate the maximum displacement as cold start condition displacement reduced to 10µm from 65µm after a period of time.
- The resultant error compensation obtained is utilized so that the error in displacement can be reduced from 75% to 98%, so the better quality of the products are established where no rejection of products are observed.

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